The Tippe Top

In classical mechanics, the movement of rigid bodies is generally described by two analogous vector equations: $F = dp/dt$ for translation of the centre of mass, and $M = dL/dt$ for rotation around the centre of mass, with $F$ the total external force, $p$ the momentum, $M$ the total moment of external forces, and $L$ the angular momentum.

We consider the intriguing movement of the tippe top. It consists of a spherical body and a cylindrical stem, with the centre of mass $CM$ displaced from the centre $c$ of the sphere (see Fig. 1). When initially put into rotation around its axis of symmetry $e_3$ vertical, the stem gradually moves downwards and finally the top flips over into a stable vertical rotation on the stem. Apparently the rotation has changed sign, while vector $L$ has preserved its original vertical position. Further, $CM$ has moved upwards at the cost of a decrease in magnitude of $L$. This unexpected behaviour is explained by the action of a friction force $F$ at the (slipping) contact point of the top with the surface (red star pointing towards the reader).

$F$ causes a moment $M$, which can be imagined to have vector components $M_{1,2}$ and $M_3$, the latter along the axis of symmetry $e_3$. Likewise, the angular momentum $L$ has components $L_{1,2}$ and $L_3$. In the beginning, $L_3 = L$ and $L_{1,2} = 0$. Then, due to instability, $F$ originates and the resulting $M_3$ tends to decrease $L_3$, while $M_{1,2}$ starts to increase $L_{1,2}$. As $L$ remains constant, the angle $\theta$ of the top's inclination will grow to fulfil proper vector addition. When $\theta = \pi/2$, $L_3 = 0$ and $L_{1,2} = L$.

![Diagram of the Tippe Top](image)

Fig. 1

Then the rotation along $e_3$ changes sign and, again through the action of $M_{1,2}$ and $M_3$, $L_3$ starts to grow at the cost of $L_{1,2}$. Finally, the stem will scrape the surface (see Fig. 2) and through the action of a new frictional force $F'$ with moment $M'$ the top will lift itself up and
strive towards a stable, though extinguishing, rotation on the stem. In fact, the component \( L_{1,2} \) is extinguished by the new \( M_{1,2} \) and \( L_3 \) finally becomes equal to \( L \).

Fig. 2

FAB, 2007-10-31